



Name:

Code no.

Answer the following questions:

(1) If $g(x) = 1 + x^2$, $f(x) = \sqrt{8 - |x|}$:

- (a) Find the composite function $g \circ f$ and determine its domain and range.
 (b) Is the function f one to one on its natural domain? Justify your answer?
 (c) Is the function $g \circ f$ is bounded? Find $\max g \circ f$, $\min g \circ f$ (if it exist)?

(2) Use the $\varepsilon - \delta$ definition to prove that $\lim_{x \rightarrow 4} \sqrt{x} = 2$, and find one of the values of δ corresponding to the value $\varepsilon = 0.05$

(3) Calculate $\lim_{x \rightarrow 0} \left(\frac{\sqrt{x+1} - 1}{\sin 2x} \right)$, and then discuss the continuity of the following function in the closed interval $[-1, 1]$:

$$f(x) = \begin{cases} \left| x - \frac{1}{4} \right| + x & , -1 \leq x \leq 0 \\ \frac{\sqrt{x+1} - 1}{\sin 2x} & , 0 < x \leq 1 \end{cases}$$

(4) Determine the true and false phrase in the following and give an example in the case when the phrase is wrong to justify your answer:

- (a) Every decreasing function is one to one.
 (b) If $f(x) = [\cos x]$ is the step function defined on $\mathbb{R}$, then its range is the set $\{-1, 0, 1\}$.
 (c) Every continuous function has a maximum and minimum value.
 (d) If the function $|f|$ is continuous at a point x_0 , then the function f is also continuous at x_0 .
 (e) The equation $x^3 - 5x + 1 = 0$ has a real root in the interval $]0, 1[$.

End of the Exam